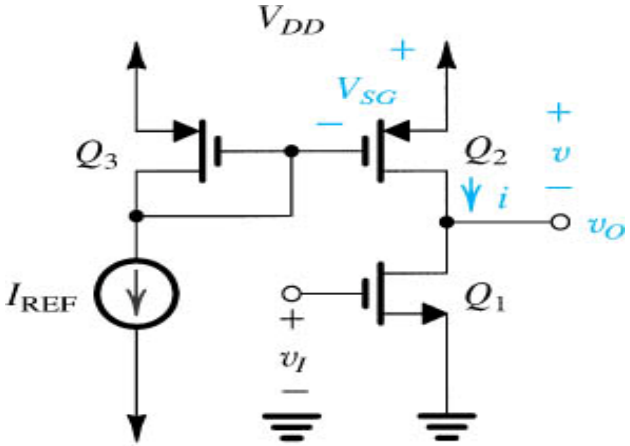
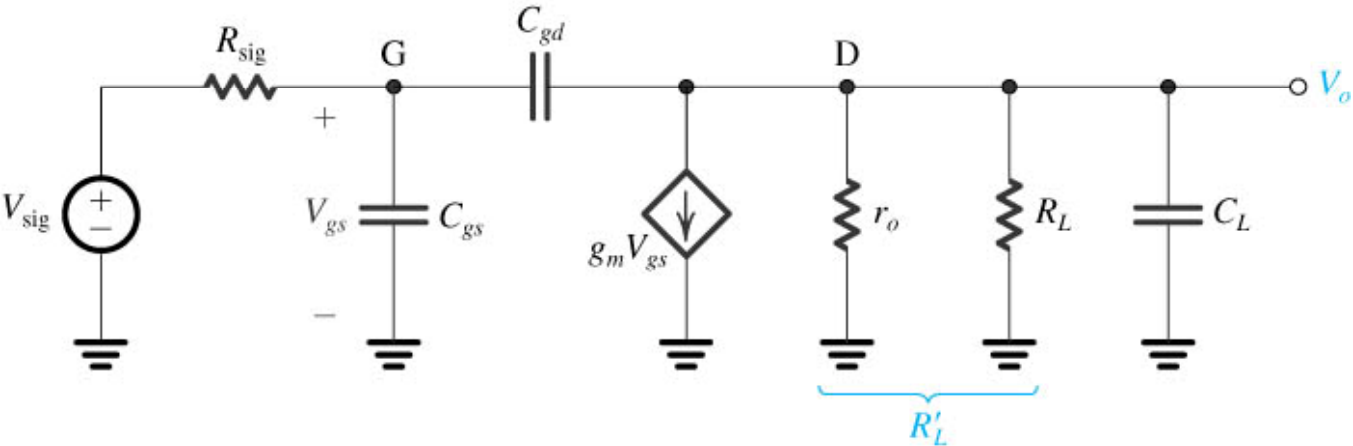


# Lect. 24: High-Frequency Response of MOSFET CS

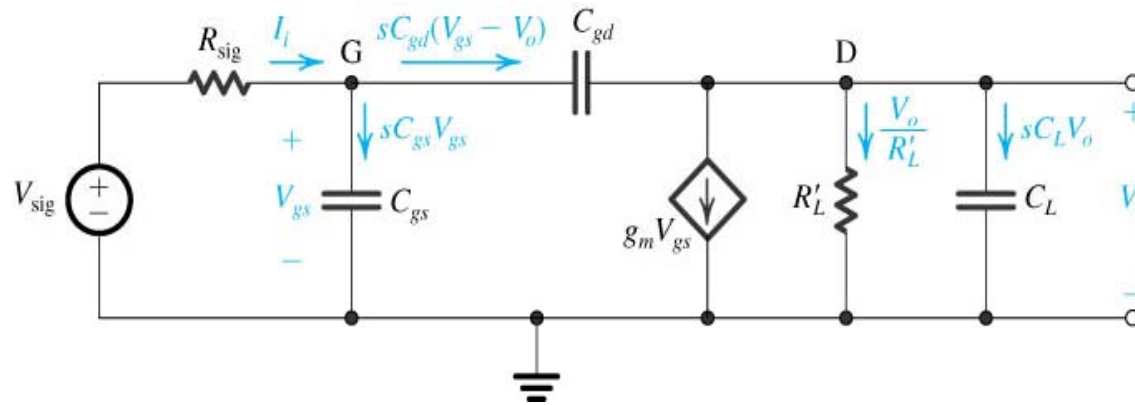


How fast can this operate?



$R_L, C_L$  due to  $Q_2$  and external load

# Lect. 24: High-Frequency Response of MOSFET CS

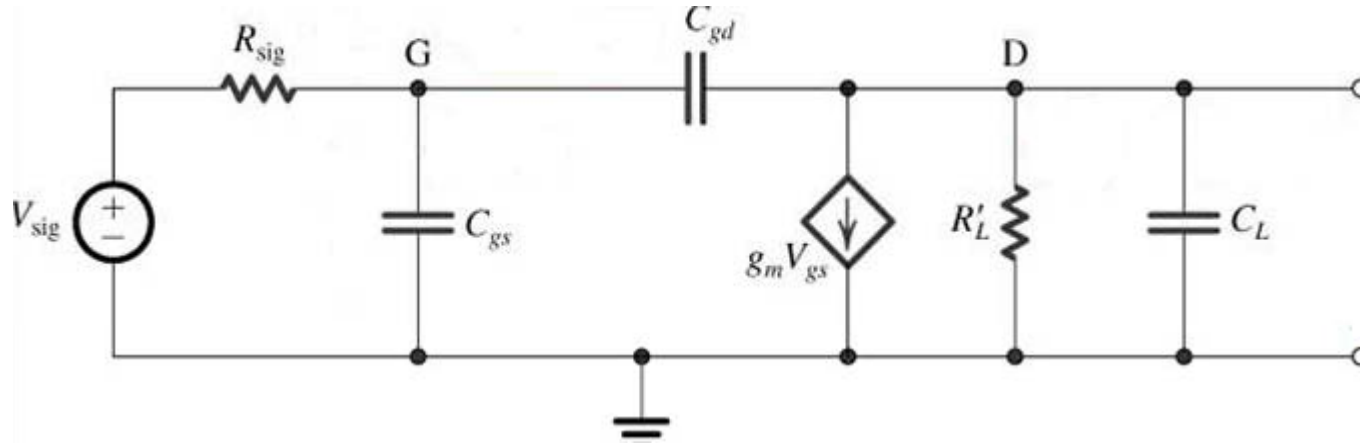


It can be shown from an exact (but complicated ) analysis that

$$\frac{V_o}{V_{sig}} = \frac{-(g_m R'_L) [1 - s(C_{gd} / g_m)]}{1 + s \left\{ [C_{gs} + C_{gd} (1 + g_m R'_L)] R_{sig} + (C_L + C_{gd}) R'_L \right\} + s^2 [(C_L + C_{gd}) C_{gs} + C_L C_{gd}] R_{sig} R'_L}$$

Too complex. A simpler way of estimating  $f_H$ , high-frequency 3-dB frequency?

# Lect. 24: High-Frequency Response of MOSFET CS

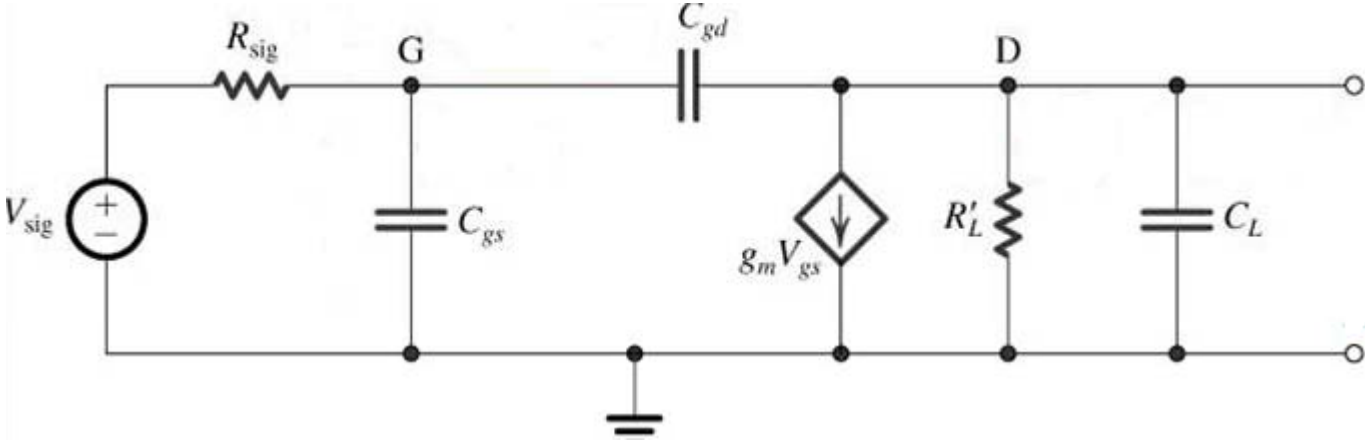


Open-Circuit Time Constant Method for approximating  $f_H$

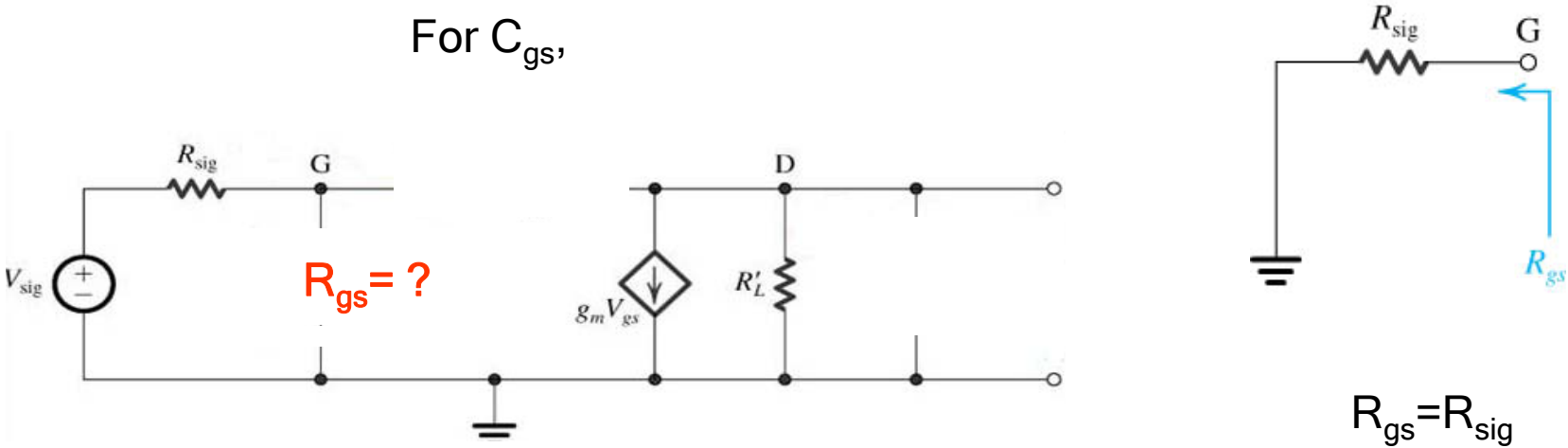
1. Select one capacitor,  $C_i$ , and set others to open.
2. Determine  $R_i$ , the resistance seen by  $C_i$ .
3. Repeat above for all capacitors.

$$\text{Then, } \omega_H \sim \frac{1}{\sum_i C_i R_i}$$

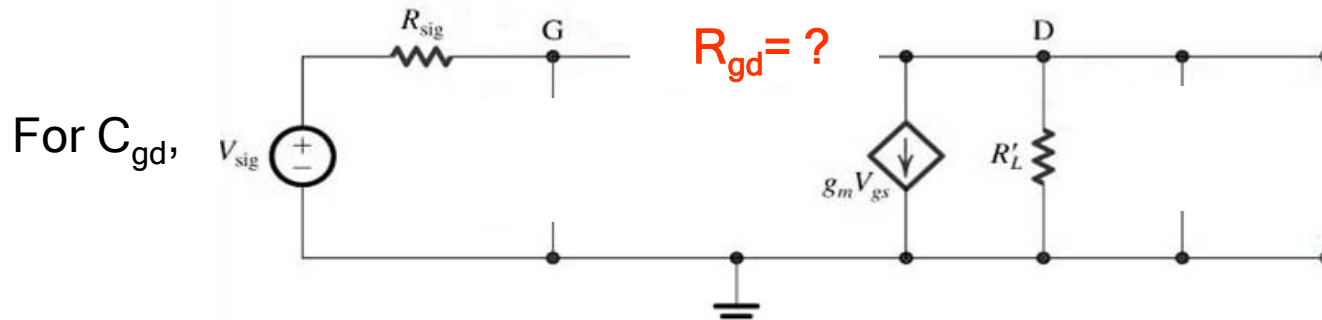
# Lect. 24: High-Frequency Response of MOSFET CS



For  $C_{gs}$ ,



# Lect. 24: High-Frequency Response of MOSFET CS



$$I_x = g_m V_{gs} + \frac{V_{gs} + V_x}{R_L'}$$

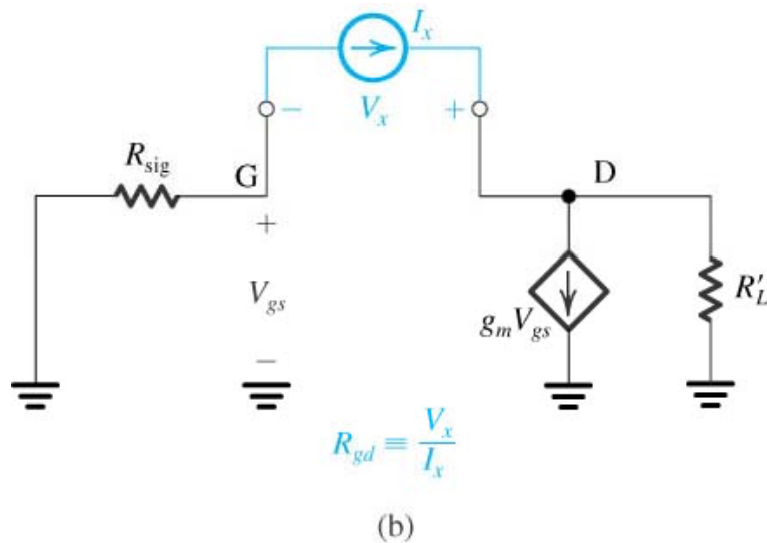
$$V_{gs} = -I_x R_{sig}$$

$$I_x = -g_m I_x R_{sig} + \frac{-I_x R_{sig} + V_x}{R_L'}$$

$$I_x \left( 1 + g_m R_{sig} + \frac{R_{sig}}{R_L'} \right) = \frac{V_x}{R_L'}$$

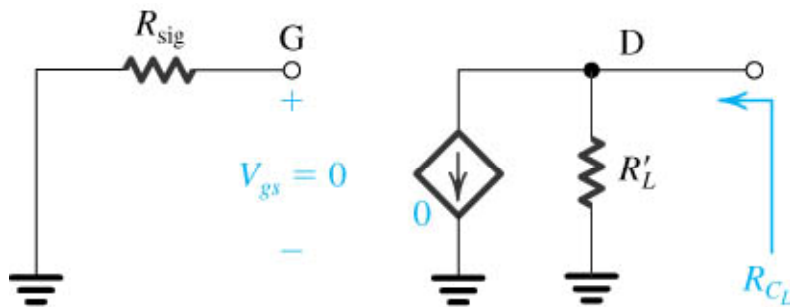
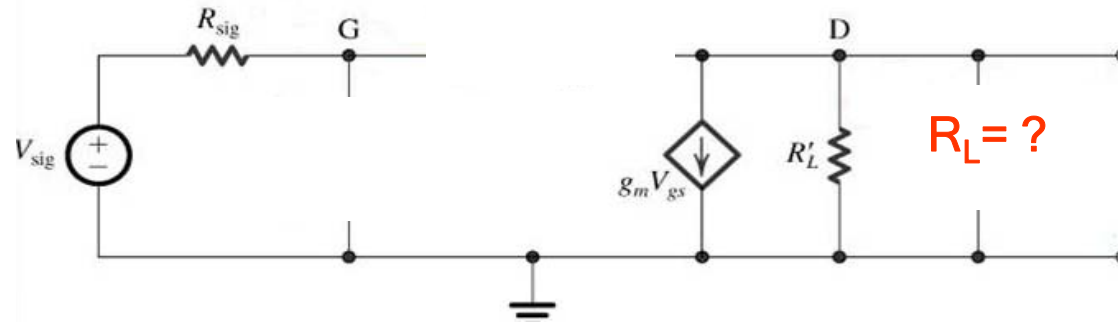
$$\frac{V_x}{I_x} = \left( 1 + g_m R_{sig} + \frac{R_{sig}}{R_L'} \right) R_L' = R_L' + g_m R_{sig} R_L' + R_{sig}$$

$$\therefore R_{gd} = R_{sig} (1 + g_m R_L') + R_L'$$



# Lect. 24: High-Frequency Response of MOSFET CS

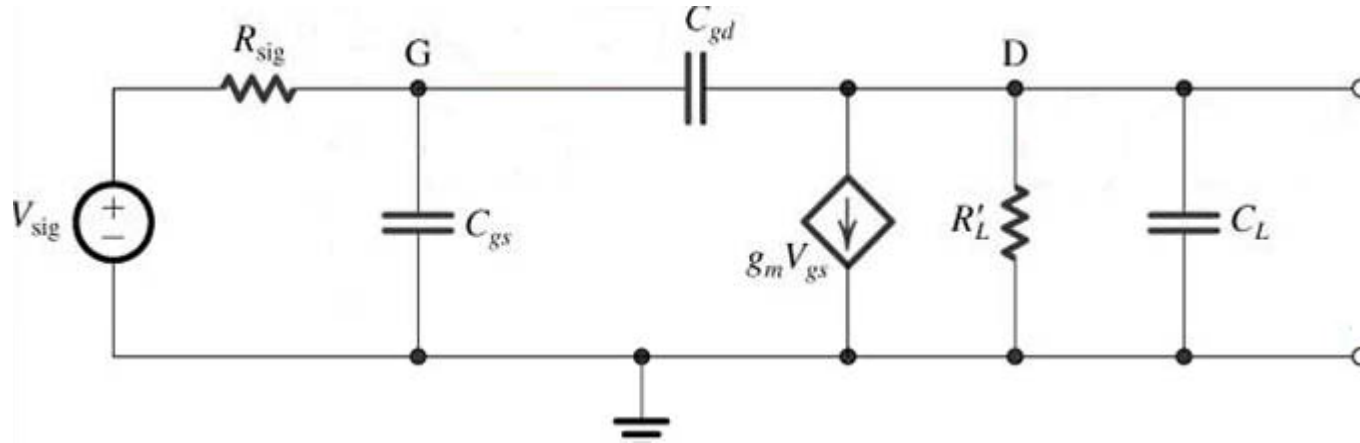
For  $C_L$ ,



$$R_L = R'_L$$

(c)

# Lect. 24: High-Frequency Response of MOSFET CS



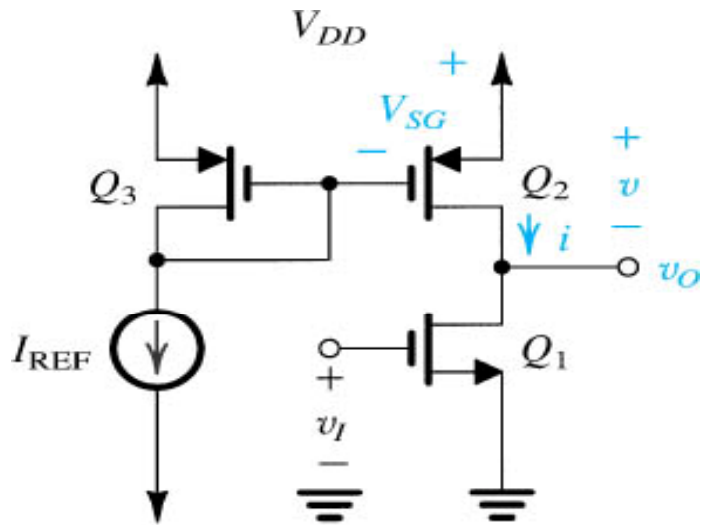
Open-Circuit Time Constant Method for approximating  $f_H$

1. Select one capacitor,  $C_i$ , and set others to open.
2. Determine  $R_i$ , the resistance seen by  $C_i$ .
3. Repeat above for all capacitors.

$$\omega_H \sim \frac{1}{\sum_i C_i R_i} = \frac{1}{\tau_H}, \text{ where } \tau_H = C_{gs} R_{sig} + C_{gd} [R_{sig} (1 + g_m R_L') + R_L'] + C_L R_L'$$

Miller Effect!

# Lect. 24: High-Frequency Response of MOSFET CS



Determine  $f_H$  for the CS shown left.

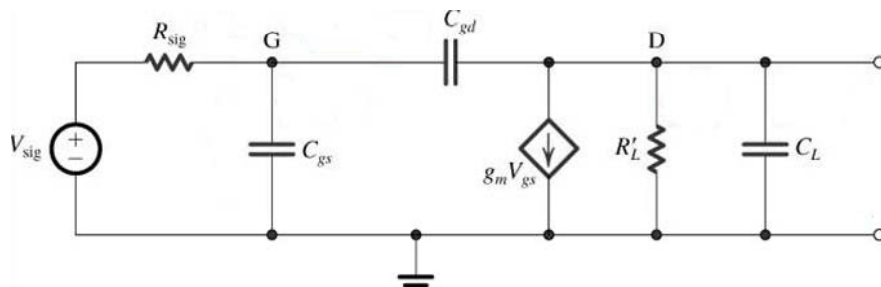
$I_{REF} = 100\mu\text{A}$ ,  $W/L = 7.2\mu\text{m}/0.36\mu\text{m}$   
 $k_n' = 387\mu\text{A}/\text{V}^2$ ,  $k_p' = 86\mu\text{A}/\text{V}^2$ ,  
 $r_{o1} = 18\text{k}\Omega$ ,  $r_{o2} = 22\text{k}\Omega$   
 $C_{gs} = 20\text{fF}$ ,  $C_{gd} = 5\text{fF}$ ,  $C_L = 25\text{fF}$ ,  $R_{sig} = 10\text{k}\Omega$ .

$$R_{gs} = R_{sig} = 10\text{k}\Omega$$

It can be shown

$$R_{gd} = R_{sig} (1 + g_m R_L') + R_L' = 142.8\text{k}\Omega$$

$$R_{C_L} = R_L' = 9.82\text{k}\Omega$$



$$\tau_{gs} = C_{gs} R_{gs} = 200\text{ps}$$

$$\tau_{gd} = C_{gd} R_{gd} = 714\text{ps}$$

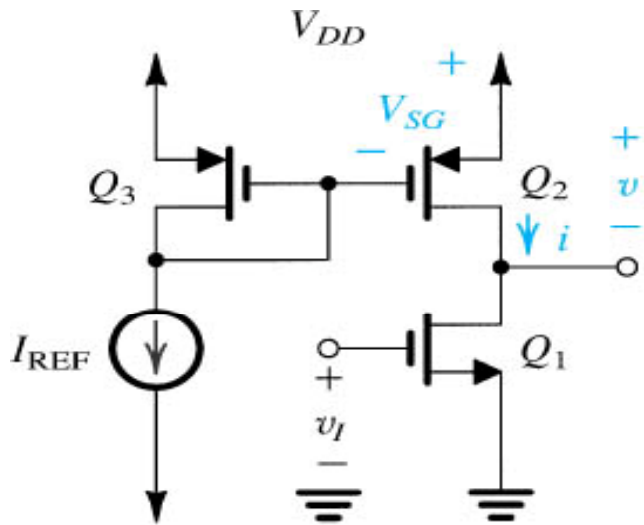
$$\tau_{C_L} = C_L R_{C_L} = 246\text{ps}$$

$$\tau_H = \tau_{gs} + \tau_{gd} + \tau_{C_L} = 1160\text{ps}$$

$$f_H = \frac{1}{2\pi\tau_H} = \frac{1}{2\pi \times 1160 \times 10^{-12}} = 137\text{MHz}$$



# Lect. 24: High-Frequency Response of MOSFET CS

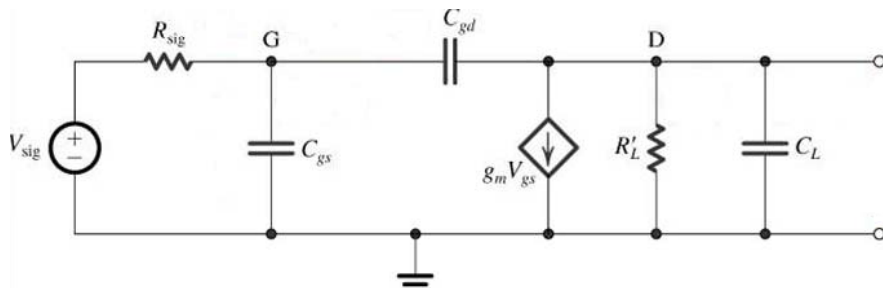


How accurate is  $f_H = 137\text{MHz}$  estimated by the open-circuit time constant method?

From the exact analysis,

$$\frac{V_o}{V_{sig}} = \frac{-(g_m R'_L)[1 - s(C_{gd}/g_m)]}{1 + s\{[C_{gs} + C_{gd}(1 + g_m R'_L)]R_{sig} + (C_L + C_{gd})R'_L\} + s^2[(C_L + C_{gd})C_{gs} + C_L C_{gd}]R_{sig} R'_L}$$

$f_H = 145.3\text{MHz}$

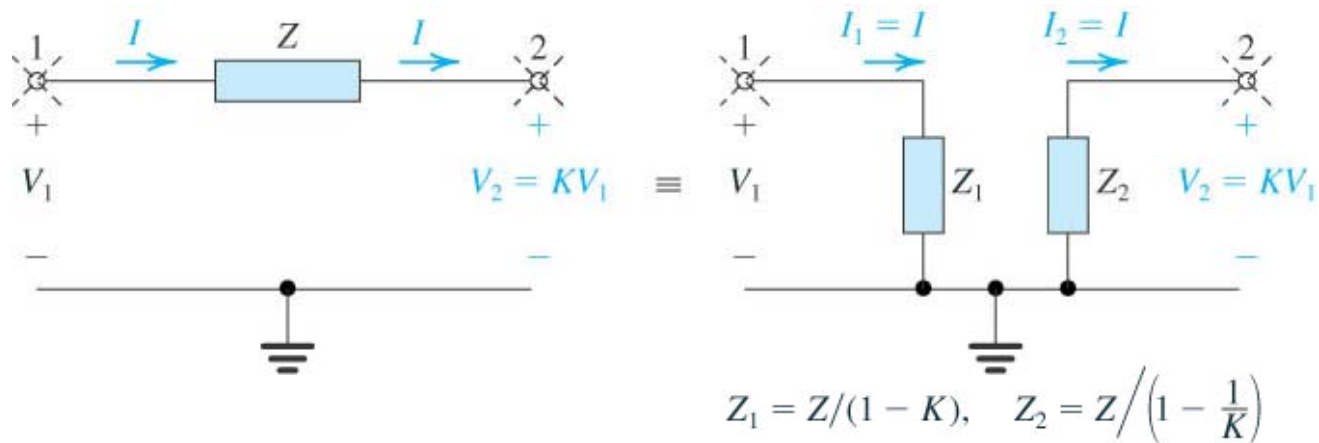


What is the most influential capacitor for  $f_H$ ?

# Lect. 24: High-Frequency Response of MOSFET CS

Miller's Theorem:

Two circuits below are identical assuming the rest of circuit does not change



$$I_1 = \frac{V_1}{Z_1} = I = \left( \frac{V_1 - KV_1}{Z} \right) \quad \text{(a)}$$

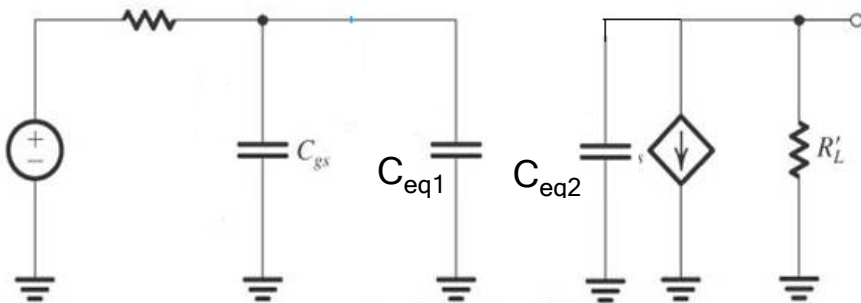
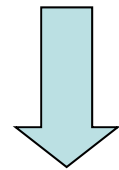
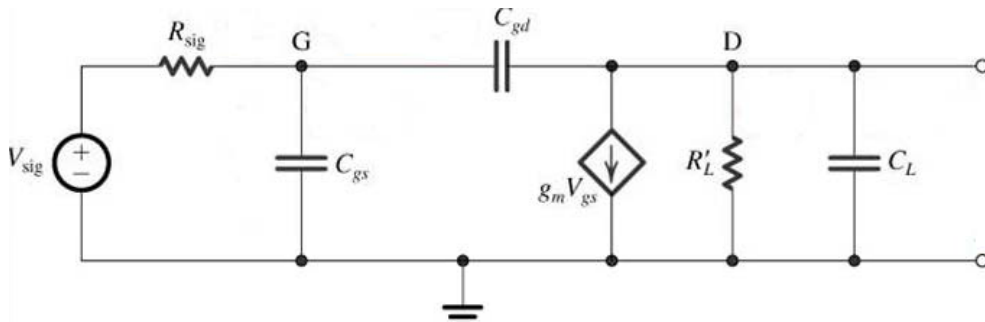
$$\therefore Z_1 = \frac{V_1 Z}{V_1 - KV_1} = \frac{Z}{1 - K}$$

$$I_2 = \frac{0 - V_2}{Z_2} = \frac{0 - KV_1}{Z_2} = I = \frac{V_1 - KV_1}{Z} \quad \text{(b)}$$

$$\therefore Z_2 = \frac{-KV_1 Z}{V_1 - KV_1} = \frac{Z}{\left(1 - \frac{1}{K}\right)}$$

# Lect. 24: High-Frequency Response of MOSFET CS

## CS amplifier



With Miller Theorem,

$$Z_1 = \frac{Z}{1-K}, \quad Z_2 = \frac{Z}{\left(1-\frac{1}{K}\right)}$$

$$\frac{1}{sC_{eq1}} = \frac{1}{sC_{gd}} \frac{1}{1-K}$$

$$\therefore C_{eq1} = C_{gd}(1-K) = C_{gd}(1+g_m R'_L)$$

$$\frac{1}{sC_{eq2}} = \frac{1}{sC_{gd}} \frac{1}{1-1/K}$$

$$\therefore C_{eq2} = C_{gd}\left(1-\frac{1}{K}\right) = C_{gd}\left(1+\frac{1}{g_m R'_L}\right)$$

For  $f_H$ , the influence of  $C_{gd}$  becomes larger by factor of  $(1+g_m R'_L)$  : Miller Effect

→ CS amplifier is slow!